The Canadian Underground Economy: An Examination of Giles and Tedds

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PRÉCIS

Dans leur ouvrage *Taxes and the Canadian Underground Economy*, David E.A Giles et Lindsay M. Tedds décrivent l'existence d'une importante économie souterraine au Canada qui connaît une croissance rapide. Ils estiment qu'elle équivaut à un montant de revenu additionnel — qui échappe aux administrations et qui n'est pas imposé — variant entre 3,46 % du PIB officiel en 1976 et 15,64 % du PIB en 1995. D'autres personnes ont contesté ces chiffres, mais la méthode économétrique utilisée par Giles et Lindsay est complexe. Dans cet article, Trevor Breusch décortique les modèles, les prévisions et les comparaisons, et révèle l'origine des résultats. Il montre que le cheminement chronologique des estimations des auteurs a peu ou pas à voir avec le revenu souterrain exprimé en pourcentage du PIB et que le niveau général de leurs estimations provient de résultats mathématiques.

ABSTRACT

In their book *Taxes and the Canadian Underground Economy*, David E.A. Giles and Lindsay M. Tedds describe a hidden economy in Canada that is large and growing rapidly. They estimate an amount of additional income—unobserved by the authorities and untaxed—ranging from a low of 3.46 percent of official GDP in 1976 to a high of 15.64 percent of GDP in 1995. Others have questioned these findings, but the econometric method employed by Giles and Tedds is complex. In this paper, Trevor Breusch peels back the separate layers of model fitting, prediction, and benchmarking and reveals the origins of their results. He shows that the time path of their estimates has little or no connection with underground income as a percentage of GDP and that the overall level of their estimates is a result of numerical accidents.

KEYWORDS: UNDERGROUND ECONOMY ■ TAX EVASION ■ ECONOMETRICS ■ MATHEMATICAL MODELS

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CONTENTS

INTRODUCTION

The first third of *Taxes and the Canadian Underground Economy* by David Giles and Lindsay Tedds[1] provides a survey of research on the phenomenon of "unrecorded" or "unreported" income, with emphasis on the estimates that have been made of its size in various countries. Particular attention is paid, naturally, to the evidence for Canada. In the central chapters of the book, the authors describe their techniques for estimating the extent of underground economic activity. They construct a time series of values from 1976 to 1995 to represent hidden income in Canada as a percentage of officially measured GDP. This series is used as data in the latter parts of the book, where the authors examine the relationship of their estimate of the underground economy to other economic variables and the consequences for taxation collection and policy.

I am concerned in this paper with the estimation technique and results of Giles and Tedds, as described in chapters 6 and 7. The basis of their method is the MIMIC (multiple indicator, multiple cause) model of Zellner[2] and Goldberger,[3] first used in a study of underground economies by Frey and Weck-Hannemann.[4] Giles[5] gives the approach new life by incorporating the modern econometric tools of cointegration and comprehensive diagnostic testing. His time series of estimates for New

1 David E.A. Giles and Lindsay M. Tedds, *Taxes and the Canadian Underground Economy*, Canadian Tax Paper no. 106 (Toronto: Canadian Tax Foundation, 2002).

2 Arnold Zelner, "Estimation of Regression Relationships Containing Unobservable Variables" (1970) vol. 11, no. 3 *International Economic Review* 441-54.

3 Arthur S. Goldberger, *Structural Equation Methods in the Social Sciences* (Amsterdam: North-Holland, 1972).

4 Bruno S. Frey and Hannelore Weck-Hannemann, "The Hidden Economy as an 'Unobserved' Variable" (1984) vol. 26, nos. 1-2 *European Economic Review* 33-53.

5 David E.A. Giles, "Modelling the Hidden Economy and the Tax-Gap in New Zealand" (1999) vol. 24, no. 4 *Empirical Economics* 621-40.

Zealand is analyzed extensively in Giles,[6] Giles and Caragata,[7] and Caragata and Giles.[8] The first application of the method to Canada is an unpublished MA thesis by Tedds;[9] the Giles and Tedds book is an extension and refinement of that earlier work. Several other papers use the estimated underground economy series from the book as data, including Giles, Tedds, and Werkneh,[10] who examine Granger causality between the underground and measured economies, and Tedds and Giles,[11] who compare Canada with New Zealand.

The MIMIC model approach to estimation of the underground economy appears to be growing in popularity. A number of authors are taking up the Giles and Tedds method: Dell'Anno and Schneider[12] construct a series for Italy and report some results for other OECD countries; Dell'Anno et al.[13] compare France, Spain, and Greece; Bajada and Schneider[14] study Australia and other nations in the Asia-Pacific; in

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6 David E.A. Giles, "Causality Between the Measured and Underground Economies in New Zealand" (1997) vol. 4, no. 1 *Applied Economics Letters* 63-67; David E.A. Giles, "The Hidden Economy and Tax-Evasion Prosecutions in New Zealand" (1997) vol. 4, no. 5 *Applied Economics Letters* 281-85; David E.A. Giles, "Testing for Asymmetry in the Measured and Underground Business Cycles in New Zealand" (1997) vol. 73, no. 222 *The Economic Record* 225-32; and David E.A. Giles, "The Rise and Fall of the New Zealand Underground Economy: Are the Responses Symmetric?" (1999) vol. 6, no. 3 *Applied Economics Letters* 185-89.

7 David E.A. Giles and Patrick J. Caragata, "The Learning Path of the Hidden Economy: The Tax Burden and Tax Evasion in New Zealand" (2001) vol. 33, no. 14 *Applied Economics* 1857-67.

8 Patrick J. Caragata and David E.A. Giles, "Simulating the Relationship Between the Hidden Economy and the Tax Size and the Tax Mix in New Zealand," in Gerald W. Scully and Patrick J. Caragata, eds., *Taxation and Limits of Government* (Boston: Kluwer Academic, 2000), 221-40.

9 Lindsay M. Tedds, "Measuring the Size of the Hidden Economy in Canada" (MA extended essay, University of Victoria, Department of Economics, 1998).

10 David E.A. Giles, Lindsay M. Tedds, and Gugsa T. Werkneh, "The Canadian Underground and Measured Economies: Granger Causality Results" (2002) vol. 34, no. 18 *Applied Economics* 2347-52.

11 Lindsay M. Tedds and David E.A. Giles, *Modelling the Underground Economies in Canada and New Zealand: A Comparative Analysis*, Econometrics Working Paper EWP 0003 (Victoria: University of Victoria, Department of Economics, 2000).

12 Roberto Dell'Anno and Friedrich Schneider, "The Shadow Economy of Italy and Other OECD Countries: What Do We Know?" *Journal of Public Finance and Public Choice* (forthcoming) (available on the Web at http://www.econ.jku.at/Schneider/ForthcomingPFPC_final%20version%2008_07.pdf).

13 Roberto Dell'Anno, Miguel Gómez, and Ángel Alañón Pardo, "Shadow Economy in Three Very Different Mediterranean Countries: France, Spain and Greece: A MIMIC Approach" (Università degli Studi di Salerno, Dipartimento di Scienze Economiche e Statistiche, April 2004) (available on the Web at http://www.unisi.it/criss/download/meeting2004/papers/dellanno.pdf).

14 Christopher Bajada and Friedrich Schneider, "The Development and Size of the Shadow Economies in the Asia-Pacific" *Pacific Economic Review* (forthcoming) (available on the Web at http://www.econ.jku.at/Schneider/ShadEcASIA2Bajadadraft1.pdf).

Schneider et al.[15] the focus is India in comparison with 18 other Asian countries; and in Schneider and Klinglmair[16] there are comparisons of dozens of countries around the world. In addition, Tedds[17] provides some updating and fine-tuning of the Canadian estimates from the book.

The idea of the MIMIC model in this context is to represent the underground economy as a latent variable or index, which has causes and effects that are observable but which cannot itself be directly measured. Thus, there are two kinds of observed variables in the analysis, one consisting of "causal" (or exogenous) variables and another of "indicator" (or endogenous) variables, which are connected by a single unobserved index. Values of the index over time are inferred from data on causes and indicators by estimating the statistical model and predicting the index. The fitted index is then interpreted as a time series of the magnitude of the underground economy. Of course, whether the interpretation is convincing will depend on the specification of the variables.

Some writers argue that the distinctions between "unrecorded" and "unreported" income are so important that it is mistaken to attempt a one-dimensional measure of *the* underground economy.[18] Even if we agree on that label for the single connection between the groups of variables in the model, we still have to establish that the index is measuring income and not some other dimension of economic transactions, such as turnover. Assuming agreement on that count, it remains to be shown that the index represents underground income measured relative to the official level of national income (the interpretation given by Giles and Tedds) rather than the absolute level of hidden income. In the latter case it is further possible to question whether the income is in nominal money value or deflated by a price index. These uncertainties in what is being measured arise because the MIMIC approach is almost entirely statistical, conducted with little or no guidance from economic reasoning.

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15 Friedrich Schneider, Kausik Chaudhuri, and Sumana Chatterjee, *The Size and Development of the Indian Shadow Economy and a Comparison with Other 18 Asian Countries: An Empirical Investigation*, Working Paper no. 0302 (Linz, Austria: Johannes Kepler University of Linz, Department of Economics, April 2003) (available on the Web at http://www.econ.jku.at/papers/2003/wp0302.pdf).

16 Friedrich Schneider and Robert Klinglmair, *Shadow Economies Around the World: What Do We Know?* Working Paper no. 0403 (Linz, Austria: Johannes Kepler University of Linz, Department of Economics, April 2004) (available on the Web at http://www.econ.jku.at/papers/2004/wp0403.pdf).

17 Lindsay M. Tedds, "The Underground Economy in Canada," in Christopher Bajada and Friedrich Schneider, eds., *Size, Causes and Consequences of the Underground Economy* (Aldershot, UK: Ashgate, forthcoming).

18 Edgar L. Feige, "A Re-Examination of the 'Underground Economy' in the United States" (1986) vol. 33, no. 4 *International Monetary Fund Staff Papers* 768-81; Paolo Caridi and Paolo Passerini, "The Underground Economy, the Demand for Currency Approach and the Analysis of Discrepancies: Some Recent European Experience" (2001) vol. 47, no. 2 *The Review of Income and Wealth* 239-50; and Organisation for Economic Co-operation and Development, *Measuring the Non-Observed Economy: A Handbook* (Paris: OECD, 2002), among others.

There are critics of the uses of MIMIC modelling to estimate the size of the underground economy. In response to Frey and Weck-Hannemann's seminal study, Helberger and Knepel[19] show that the results are unstable in the face of minor changes in either the data period or the list of countries studied. They also argue that the lists of causal and indicator variables are unconvincing for the purpose, and that other aspects of the econometric methods are unsatisfactory.

When the Giles and Tedds book was published, the *Canadian Tax Journal* carried an introductory note from the journal editor and two papers of scholarly criticism by Smith[20] and Hill,[21] together with a rejoinder by Giles and Tedds.[22] These critics express doubts about the methodology used in the book, especially the complexity of the multi-layer estimation strategy. In an echo of the Helberger and Knepel critique, they question the relevance of the causal and indicator variables selected by Giles and Tedds. They also indicate how the results of such economy-wide investigations are of limited use to policy makers. The matters I examine in this paper are more specific to the estimation method of Giles and Tedds and somewhat more technical than the ones canvassed in the reviews by Smith and Hill.

It will be seen in what follows that there is almost no difference between the measure of the underground economy calculated by Giles and Tedds and just one of their causal variables. The time-series properties of that single variable dictate almost all of their substantive findings about change over time. The variable concerned is a component of national income, and it measures the absolute level of economic activity in a sector, not activity in proportion to measured GDP. Further, the variable is measured in nominal Canadian dollars, since it is not adjusted for inflation in the price level. These findings strain the credibility of the interpretation they give their estimates.

The MIMIC model in the Giles and Tedds approach provides a raw index, which gives only the relative movement in their estimated time series from year to year. The analogy is to a price index, which captures the relative proportional movements over time in the quantity of interest, but which is conventionally scaled to 100 in some base period. In MIMIC modelling the scale of the index is similarly arbitrary, but Giles and Tedds seek to interpret the series as tracking the size of the underground economy. To this effect they specify and estimate a currency demand model, from which they produce auxiliary estimates of the phenomenon of interest. They use these other estimates to calibrate or benchmark the raw index from the MIMIC

19 Christof Helberger and Helmut Knepel, "How Big Is the Shadow Economy? A Re-Analysis of the Unobserved-Variable Approach of B.S. Frey and H. Weck-Hannemann" (1988) vol. 32, no. 4 *European Economic Review* 965-76.

20 Roger S. Smith, "The Underground Economy: Guidance for Policy Makers?" (2002) vol. 50, no. 5 *Canadian Tax Journal* 1655-61.

21 Roderick Hill, "The Underground Economy in Canada: Boom or Bust?" (2002) vol. 50, no. 5 *Canadian Tax Journal* 1641-54.

22 David E.A. Giles and Lindsay M. Tedds, "Response" (2002) vol. 50, no. 5 *Canadian Tax Journal* 1662-67.

model. They interpret the resulting scaled series as representing underground economy income as a percentage of measured GDP.

I show that the currency model used by Giles and Tedds for benchmarking is unidentified—exactly and globally. Whatever values the parameters might take, there are always some other values with different implications but exactly the same observable consequences. No amount of any conceivable data can distinguish among these observationally equivalent structures. This is a logical problem with the form of the model, and it has profound implications for inference. Fatally for the present purpose, the parameters that determine the benchmark are among the ones that are indeterminate in the model.

Because of the identification failure, the benchmark estimate comes from numerical accidents in the estimation process. I show how radically different estimates are obtained when minor changes are made to the starting values of the estimation algorithm or even when two different, but well-established, software packages are employed for estimation of the same model. Similar problems may be expected when even minor variations are made to the model or in the data, such as when variables or observations are added or dropped. The implication is that the overall level of the series estimated by Giles and Tedds is a mirage.[23]

THE RESULTS AND ESTIMATION STRATEGY

The Giles and Tedds estimate of the underground economy in Canada is a 20-year time series that ranges from a low of 3.46 percent of official GDP in 1976 to a high of 15.64 percent of GDP in 1995, passing through the benchmark value of 9.45 percent in 1986.[24] This series is plotted herein in figure 1. The detailed time shape of the series will be important in the next section, where I consider the predictions of the MIMIC model, but for the moment I will concentrate on the two prominent features that are highlighted with arrows.

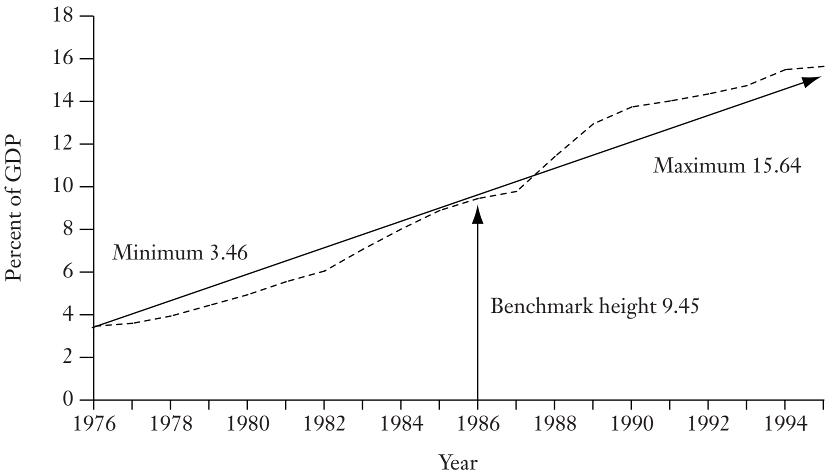
First is the strong growth indicated by the sloping arrow. The series increases by a multiple of 4.5 times over 20 years, which is equivalent to a compound rate of increase of 7.8 percent per annum. This phenomenal growth is more remarkable for being relative to observed GDP. Over the same period, recorded Canadian real GDP grew by 64 percent. Thus it seems that the level of underground income in Canada increased in real terms[25] more than seven times over while the observed

23 Smith, supra note 20, at 1656, observes in reference to the currency model: "The particular model they use is subject to many of the same criticisms levied against other models—for example, that minor changes may lead to substantially different results." No further evidence is offered, so this seems to be a general comment about a class of models, or perhaps about models in general, and not a demonstrated defect of this particular model.

24 The time series is listed in Giles and Tedds, supra note 1, at 120, table 7.1.

25 This calculation assumes that the Giles and Tedds index is the relationship of real underground income to real measured GDP, or if the relationship is in nominal variables, that the implicit GDP price deflator applies to underground economy income.

FIGURE 1 Canadian Underground Economy, 1976-1995



Source: David E.A. Giles and Lindsay M. Tedds, *Taxes and the Canadian Underground Economy*, Canadian Tax Paper no. 106 (Toronto: Canadian Tax Foundation, 2002), 120, table 7.1.

economy much less than doubled in size. To achieve that increase in 20 years requires a compound annual rate of real growth in the underground economy of 10.5 percent.[26] This rate is unrealistic. As will be seen, these growth properties of the underground economy are derived entirely from fitting and predicting the MIMIC model.

The second important feature is the benchmark value of 9.45 percent of GDP for 1986, which is marked with the vertical arrow in figure 1. Whether this number is high or low is a matter for debate, but Giles and Tedds cite other studies that produce estimates of similar magnitude. We will see that the overall magnitude of the underground economy estimate is derived from the currency demand model.

In the MIMIC model the scalar latent variable, η_t, is written as a linear combination of a $q \times 1$ vector of causal or exogenous variables, x_t,

$$\eta_t = \gamma' x_t + \zeta_t \quad t = 1, \ldots, N, \tag{1}$$

where γ is a vector of unknown coefficients and ζ_t is an unobserved error term with zero mean and constant variance. There are other parts to the model involving the indicator variables, but for the present purpose all that matters is the insight that,

26 Growth in the index is sometimes much higher than average, to the extent that the level of real underground income increases by over 20 percent in some years.

given the estimate of the coefficient vector, the estimated index is obtained as the prediction

$$\hat{\eta}_t = \hat{\gamma}' x_t \quad t = 1, \ldots, N. \tag{2}$$

The reason for mentioning an otherwise trivial and obvious point is to flag the point that the prediction formula in (2) is not used by Giles and Tedds. Rather, they obtain the prediction as

$$\hat{\eta}_t = \hat{\gamma}' x_t^* \quad t = 1, \ldots, N, \tag{3}$$

where x_t^* is a *transformed* version of x_t.

What is the transformation? To address the obviously trending nature of the data on the causal and indicator variables, and to protect the estimation results against a charge of spurious regression, they difference the data to stationarity before fitting the model. The prediction stage, however, is based on the actual values of the variables (the levels), so that the x_t^* variables in our notation are integrated versions of the x_t variables. Of course, the time series properties of the integrated variables are very different from those of the differenced variables (especially so, since one of the variables is a component of Canadian national income in nominal dollars, and has to be differenced twice to make it stationary).

In equation 1 the dependent variable η_t is unobserved, and both γ and var(ζ_t) are parameters to be determined in estimation. Hence there is nothing in the equation to determine the scale of the unobserved index. The convention in MIMIC modelling is to relate the index to one of the indicator variables. Giles and Tedds follow this convention temporarily, and then reset the scale from the results of the currency demand modelling. From this other model they derive another time series of the underground economy. They use

> the average value of approximately 9.45 percent of measured GDP as a representative estimate of the ratio over the sample period. . . .
>
> Since the sample mean value of measured GDP occurs in 1986, we set the ratio of underground GDP to measured GDP to 9.45 percent for that year in order to convert the index time-path for the hidden economy that emerged from our MIMIC-model analysis.[27]

Thus the index in 1986 has the value 9.45 derived from the currency demand model, as shown by the vertical arrow in figure 1. The rest of the benchmarked series is given by the scaling operation

$$index_t = 9.45 \times \hat{\eta}_t / \hat{\eta}_{1986} \quad \text{for all } t = 1976, \ldots, 1995. \tag{4}$$

This confirms my earlier observation that the overall magnitude of the underground economy series is derived from the currency demand model.

27 Supra note 1, at 138.

A simple but important property of the benchmarking process is its preservation of the relativities between the measurements in different years, so that

$$index_t / index_s = \hat{\eta}_t / \hat{\eta}_s \quad \text{for all } t \text{ and } s. \tag{5}$$

This confirms the earlier observation that the proportional growth properties of the scaled index are derived entirely from fitting and predicting the MIMIC model. I represent this feature by the sloping arrow in figure 1, showing the growth of the (scaled) index between 1976 and 1995.

I am able to replicate the results of Giles and Tedds using the estimates of the coefficient vector ($\hat{\gamma}$) as published and the data on the causal variables (the components of x_t^*) supplied to me by Lindsay Tedds.[28] With these inputs in formulas 3 and 4, I get a series that is almost identical to the published index.[29]

MIMIC MODEL PREDICTION

What is the effect of predicting from the MIMIC model with the data in levels when the model has been estimated with the data in differences (in one case differenced twice)? Again, I use the $\hat{\gamma}$ coefficient estimates as published, shown as the first row of table 1, labelled "Gamma." In the next three rows of the table are some summary statistics of the causal variables, x_t^*. The two rows with obvious labels contain the mean and standard deviation of the variable over the 20 years. The row labelled "Growth" is the ratio of the variable in 1995 to its value in 1976, thus showing the growth multiple in 20 years.

For the moment it is best to set aside the meaning of the variables (details can be found in table A1) and just focus on construction of the raw index as a prediction. A feature of the summary statistics is the enormous difference in scale between the first three variables (particularly the first two) and the remaining four. Whether scale is measured in means or standard deviations, these variables are of very different orders of magnitude. Any linear combination with similar-sized coefficients will be dominated almost exclusively by the first two or three variables.[30] A second

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28 Coefficient estimates of the "preferred model" are given as model 6 in Giles and Tedds, ibid., at 130, table 7.4, and shown in the first row of table 1 herein. The data as supplied are listed in the appendix, together with the names of the relevant variable from Giles and Tedds, ibid., at 129, table 7.2.

29 In my replication of the scaled index, I get a range from 3.45 to 15.71, compared to the range 3.46 to 15.64 in the published results. This divergence is small enough to be explicable by my use of the published coefficients of the MIMIC model, which are rounded to three significant digits. While I can replicate the index using the published coefficients and the supplied data, I have been unable to replicate estimation of the MIMIC coefficients; hence my use of the published coefficients throughout.

30 Indeed, the published coefficients not only prove impossible for me to replicate, on this evidence they seem implausible. The variables in the model (even after differencing) are enormously different in scale—their means and standard deviations differ by factors in the millions—but the coefficients are all nicely in the range 0.028 to 0.627.

TABLE 1 Selected Properties of the "Causal" Variables

Variable[a]	MULT	SELF	INC	ERTE	CORP	IND	UNEM
Gamma	0.627	0.359	0.028	0.293	0.268	0.224	0.308
Mean	4.66E+05	9.68E+06	2.55E+04	1.23	2.99	12.86	9.29
Std. dev.	1.50E+05	4.56E+06	4.34E+02	0.10	0.53	0.75	1.56
Growth	3.17	4.71	1.00	1.39	0.72	1.09	1.32
Share	9.42%	90.53%	0.05%	0.00%	0.00%	0.00%	0.00%

a See table A1 for an explanation of the variables.

Source: David E.A. Giles and Lindsay M. Tedds, *Taxes and the Canadian Underground Economy*, Canadian Tax Paper no. 106 (Toronto: Canadian Tax Foundation, 2002), 120, table 7.1 and 130, table 7.4; and table A2 of this article.

feature of note is that only the first two variables have growth factors anything like the value of 4.5 found in the Giles and Tedds estimate of the underground economy in Canada. Even then, the factor of 3.17 in MULT falls well short, so we deduce that SELF has to be an important driver in the index series.

We can use the analogy with a price index to assess the relative importance of the variables in forming the prediction from the MIMIC model.[31] The predictor in expression 3 above is equivalent to a Laspeyres price index, with a price vector in each period of x_t^* and base period quantities in the vector $\hat{\gamma}$. Scaled in the conventional way to 100 in the base period (indicated by $t = 0$), the index is

$$L_t = 100 \times \left(\Sigma_{j=1}^{7} \hat{\gamma}_j x_{jt}^*\right) \Big/ \left(\Sigma_{j=1}^{7} \hat{\gamma}_j x_{j0}^*\right) \quad \text{for } t = 1976, \ \ldots, \ 1995. \tag{6}$$

Then, using the usual tool kit from index number theory, we can write this expression as a weighted combination,

$$L_t = 100 \times \Sigma_{j=1}^{7} \left(x_{jt}^* / x_{j0}^*\right) S_{jt} \quad \text{for } t = 1976, \ \ldots, \ 1995, \tag{7}$$

where the relative "prices" over time are the growth factors x_{jt}^* / x_{j0}^* and the weights are the base-period "budget shares,"

$$S_{kt} = \hat{\gamma}_k x_{k0}^* \Big/ \left(\Sigma_{j=1}^{7} \hat{\gamma}_j x_{j0}^*\right) \quad \text{for } k = 1, \ \ldots, \ 7. \tag{8}$$

Thus the row of table 1 labelled "Growth" can also be interpreted as the growth ratios of the seven individual variables that are to be combined into a single index, measured from a base in 1976 to the end of the period in 1995. The corresponding

31 Luckily, the variables and their coefficients are always positive; this need not be so in a MIMIC model, but it is needed in the analogy with a price index.

base-period weights or budget shares are shown in the last row of table 1, labelled "Share." Here we see that more that 90 percent of the movement in the index can be attributed to the variable SELF. Of what remains, almost all is due to MULT, and these two variables together account for 99.95 percent of the index. The very small amount not explained by SELF and MULT is almost entirely due to the third variable, INC. The contribution from variables four through seven in the list is effectively zero.

Before asking what these variables measure and how that affects the interpretation given to the index by Giles and Tedds, there is another way to show the dominance of SELF in forming the index. Suppose SELF alone is the raw index, and consider what happens if this variable is benchmarked in the same way as the prediction from the MIMIC model and then presented as the estimate of the underground economy. The simple correlation between SELF and the index of Giles and Tedds is 0.9999. It might be argued that this correlation is inflated by the trends in both variables, but the correlation between the first differences is also very high at 0.9922. Even the second differences series, which in the case of SELF is stationary according to Giles and Tedds, have a correlation coefficient of 0.9858.

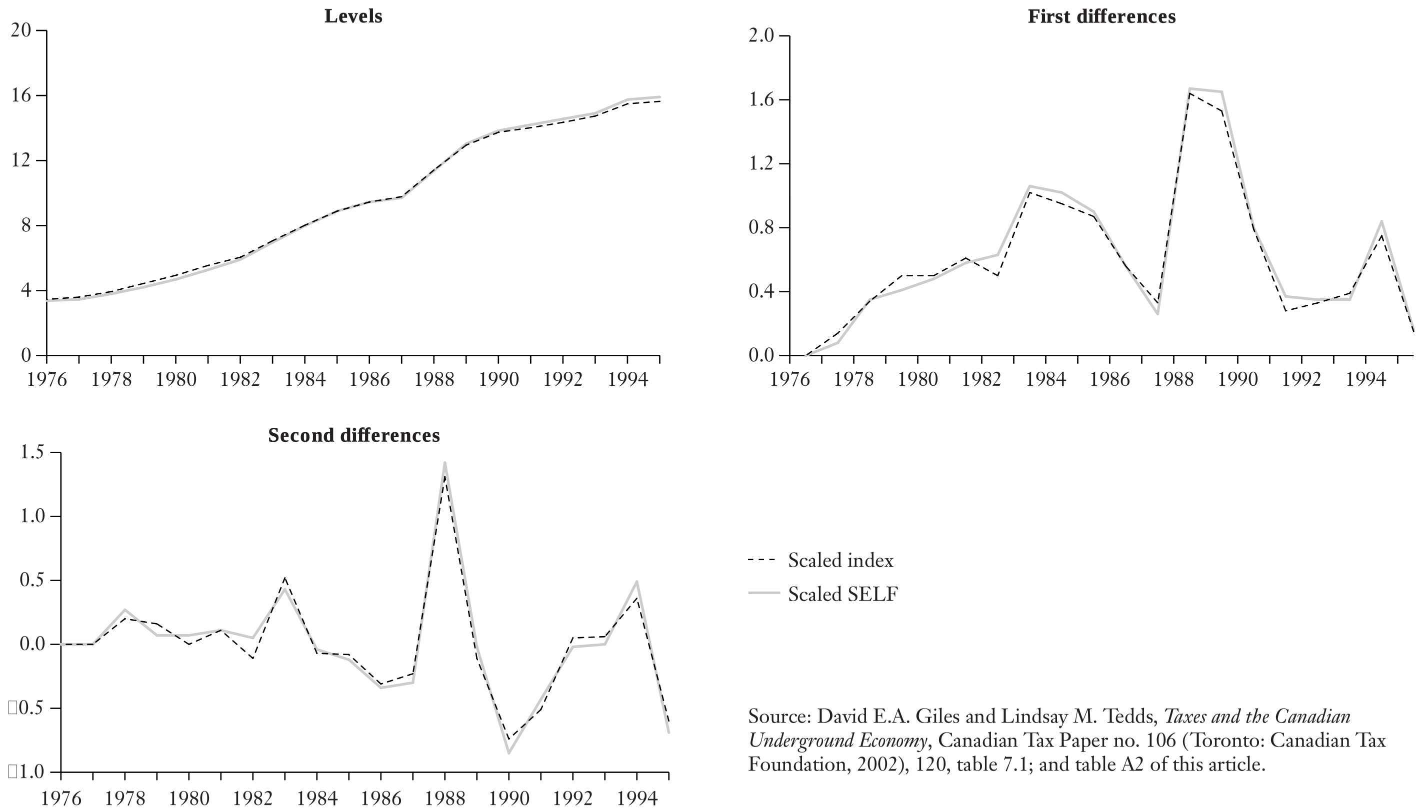
Comparisons by correlation may overstate the similarity between the variables as descriptions of a time path. Correlations are invariant to changes in both location and scale in the variables, but in forming an index series only the scale is freely adjusted. A better time-series comparison for this purpose can be seen in the plot points of figure 2. Here SELF has been scaled to the benchmark value of 9.45 in 1986, in exactly the same treatment as the scaled index. It is again obvious that the two series are almost identical. The main distinction is the slightly flatter overall slope of the index compared to SELF; this is mostly due to the minor contribution made to the index by MULT. Further, the plot points of the first differences of the two variables nearly coincide, except the index is slightly below SELF on average. Even the second differences are almost coincident throughout the period. It is clear that SELF drives almost all of the behaviour of the index apart from the level (which is set by the scaling).

What are the variables that constitute the index of Giles and Tedds? For convenience, descriptions of the relevant variables are given in table A1.[32] The dominant influence, SELF, is described as "Income of self-employed persons." As noted by Hill,[33] this variable is an economy-wide aggregate and measured in nominal dollars. SELF increases by a multiple of 4.71 over the 20 years, and we have seen that this variable drives the growth in the index almost on its own. Over the same period, nominal GDP in Canada rises by a multiple of 3.92, so that the income of the self-employed grows only slightly in relation to GDP, by a multiple of 4.71/3.92 = 1.20. The growth multiple in nominal GDP is composed of a component factor in real GDP of 1.64 and one in prices of 2.39. Expressed as changes, the income of self-employed

32 Extracted from Giles and Tedds, supra note 1, at 129, table 7.2.

33 Supra note 21.

FIGURE 2 Scaled Index and SELF



Source: David E.A. Giles and Lindsay M. Tedds, *Taxes and the Canadian Underground Economy*, Canadian Tax Paper no. 106 (Toronto: Canadian Tax Foundation, 2002), 120, table 7.1; and table A2 of this article.

persons rises by 20 percent relative to income generally, real national income goes up by 64 percent, and price inflation accounts for the remaining 139 percent of the recorded growth in SELF. Thus, most of the high level of growth in this dominant "cause" of the underground economy comes from price inflation, a further substantial part is movement in concert with the general expansion of the Canadian economy as measured by real GDP, and only a small part is due to the contributions of the self-employed relative to the economy as a whole. It is quite implausible that it charts the growth of the underground economy as a percentage of measured GDP.

The only other variable of note in the index is MULT, described as "The total number of male holders of multiple jobs aged 15 or more." Again, this is another national aggregate, not a proportional factor in measured GDP, although this time it is a real variable and not one in nominal dollars. Hill describes the selection of the causal and indicator variables in the MIMIC model as "somewhat ad hoc,"[34] an assessment I would endorse were it not for the understatement. I can add the rider that the only variables making a significant contribution to the index of Giles and Tedds are ones that, in measurement terms, are most inappropriate. Other causal variables in the model are more plausible as drivers of an underground economy (for instance, CORP and IND, which are ratios of taxes to GDP), but they are irrelevant to the calculation of the index.

This finding that SELF alone determines most of the time profile of the index, with just a little help from MULT, neatly explains the "robustness" that Giles and Tedds promote as a virtue:

> We considered about a dozen different MIMIC-model specifications, and we found that the results were extremely "robust"; that is to say, the time-paths for the underground economy yielded as "predictions" by the models were all closely similar—for the most part, indeed, virtually indistinguishable.[35]

The one feature common to all 10 different MIMIC models reported in their table 7.4 is inclusion of SELF and MULT as causal variables.

I note here that Giles and Tedds are wrong in describing the relative importance of individual variables (a description echoed by Hill in his comments on their specification). They say,

> the weight for each of the other variables indicates its importance as an indicator or a cause. . . . Self-employment is somewhat more important than either the exchange rate or unemployment. Corporate and indirect taxes together account for about 23 percent of the overall weight of the causes. The holding of more than one job is a particularly important factor; it accounts for nearly 30 percent of the total weight.[36]

34 Ibid., at 1644.

35 Supra note 1, at 115 (quotation marks in original).

36 Ibid., at 116.

From the numbers being quoted, it is clear that their calculation of a weight is the individual coefficient in proportion to the sum of the coefficients. Thus, in contrast to the shares described in 8 above, their interpretation of the weights is

$$W_k = \hat{\gamma}_k \Big/ \Sigma_{j=1}^{7} \hat{\gamma}_j \quad \text{for } k = 1, \ \dots\ , 7. \tag{9}$$

This interpretation does not make any sense. In a MIMIC model, as in simple linear regression, the magnitude of a coefficient is a function of the units in which its variable is measured. When a variable x_{jt} is rescaled one way or another, its coefficient γ_j is scaled by precisely the inverse transformation. Hence a coefficient can be made arbitrarily large or small by transforming the corresponding variable to suit. Simply quoting the relative size of a coefficient says nothing about the relative importance of that variable in a linear combination—unless the data are transformed to make all the variables essentially equal.

Perhaps some further transformations are applied to the data before the MIMIC model is estimated to obtain the published coefficients (in addition to the differencing that is documented). One possibility is that the variables are standardized by transforming them individually to zero mean and unit standard deviation. Such additional transformations are not only undocumented, they are another wedge between the x_t variables used in estimation and the x_t^* variables used in prediction. In particular, if the estimation variables are standardized, the estimated coefficients will be invariant to the units of measurement in the original variables. But when the index is formed by applying these coefficients to the original variables—as I have shown by replication is the procedure—the calculation will be quite sensitive to the units in which the variables are measured. Different substantive answers will be obtained, for instance, if SELF is measured in units of millions of dollars (as GDP is measured in the data) rather than what seems to be thousands of dollars. But this speculation about the estimation procedure does not change the properties of the index. The calculation of the relative importance of each variable to the index must be made from the actual coefficients and variables that are used in prediction, as I have done.[37]

CURRENCY DEMAND MODEL

While the time shape of the estimates of the underground economy comes from predicting the MIMIC model, the overall level of the estimates is set by benchmarking from a separate currency demand model. Giles and Tedds specify the estimation model for currency as follows.[38] Demand for (nominal) currency is written as

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37 Standardization might explain the puzzling fact that the variables are so different in magnitude while the reported coefficients are all nicely within a small range. Indeed, my replication attempts are much closer to the reported results when all the variables in the estimation model are standardized. Nonetheless, the reported results cannot be replicated, which suggests that there may be some other undocumented data transformation.

38 Giles and Tedds, supra note 1, at 109-10 and 134-36.

$$M_t = b_0\left(Y_{Rt} + Y_{Ht}\right)^{b_1} R_t^{b_2} P_t^{b_3} e_t, \tag{6.4}$$

where Y_{Rt} and Y_{Ht} are "recorded" and "hidden" real income (or output), respectively; R_t is the interest rate; P_t is the price level; and e_t is an error term.[39] Some assumptions are required to represent hidden income Y_{Ht} as a function of observable variables. After considering a longer distributed lag relationship between hidden and recorded income, and setting some interim lag coefficients to zero based on initial testing, they arrive at the relationship[40]

$$Y_{Ht} = a_0 Y_{Rt} + a_2 Y_{Rt-2}. \tag{10}$$

Substituting 10 into 6.4, and linearizing by taking logarithms, yields the "preferred" form of the model

$$\log M_t = \log b_0 + b_1 \log\left(Y_{Rt} + a_0 Y_{Rt} + a_2 Y_{Rt-2}\right) + b_2 \log R_t + b_3 \log P_t + v_t, \tag{7.7}$$

where $v_t = \log e_t$ is $NID(0, \sigma^2)$. The model they estimate is an "improved" specification, containing an additional lag of the (log) interest rate; thus

$$m_t = b_0^* + b_1 \log\left[(1 + a_0) Y_{Rt} + a_2 Y_{Rt-2}\right] + b_2 r_t + b_3 p_t + b_4 r_{t-1} + v_t, \tag{7.8}$$

where $b_0^* = \log b_0$ (I have adopted the shorthand notation of lower-case letters for logarithms of variables). Estimates by maximum likelihood (equivalent to nonlinear least squares) are reported in Giles and Tedds.[41] For convenient reference, these same results are listed in column 1 of my table 2. With a minor exception to be noted below, I have been able to replicate these results exactly using the data in the appendix.

Giles and Tedds discuss some antecedents in the literature of this form of currency demand model, but they give no indication of expected values of the parameters. I note that $b_0 > 0$ is required for the log representation of the intercept, but beyond that nothing much can be said about that parameter. The exponents in 6.4 are elasticities of currency demand with respect to real income, the interest rate, and the price level, so that we can expect $b_1 > 0$, $b_2 < 0$, and $b_3 > 0$ on the usual

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39 My equations with numbering of the form 6.n and 7.n are taken directly from Giles and Tedds. The representation of the interest rate, R, as a multiplicative factor in the demand function is a questionable practice. An interest rate usually arises either in the exponent of a continuous time theory model or, equivalently, as a markup term in a discount factor in a discrete time model. When the equation is then linearized by taking logarithms, the result is an additive term containing either the interest rate R itself or $\log(1 + R) \approx R$. A currency demand model for Canada with the better treatment of the interest rate is found in Roderick Hill and Muhammed Kabir, "Currency Demand and the Growth of the Underground Economy in Canada, 1991-1995" (2000) vol. 32, no. 2 *Applied Economics* 183-92.

40 Expression 10 is not a numbered display equation in Giles and Tedds.

41 Supra note 1, at 138, table 7.7.

reasoning for money demand models. Homogeneity of money demand in nominal income at a constant interest rate suggests $b_1 = b_3 = 1$ might hold or at least might be an interesting hypothesis to explore. In the estimation model 7.8, the interest rate elasticity is b_2 in the short run and $b_2 + b_4$ in the long run. It seems likely that both $b_2 < 0$ and $b_4 < 0$, although these are fairly weak predictions compared with the stronger one that $b_2 + b_4 < 0$.

Parameters a_0 and a_2 represent the current-period multiplier and the two-quarter-lag multiplier, respectively, of hidden income in response to recorded income. It is unclear what signs these parameters might have individually, because there is no explanation given for the relationship. There may be substitutions between sectors as well as co-movement of the two sectors, and the behaviours may be different in the short run and the long run. If the recorded part of the real economy is static, so that $Y_{Rt} = Y_{Rt-2}$, then the ratio of hidden to recorded income is $a_0 + a_2$, which is constant and must be non-negative. On the other hand, if the recorded real economy is growing steadily at a rate of g per annum, the ratio of hidden to recorded income is

$$Y_{Ht}/Y_{Rt} = \left[a_0 + a_2(1+g)^{-2}\right], \tag{11}$$

which is again constant. In this case, the non-negativity requirement on hidden income imposes a restriction on a_0 and a_2 that involves the growth rate.

With these prior requirements on the signs of the parameters in 7.8, we see that the reported estimates are mostly quite plausible. The b_0 estimate reported in Giles and Tedds is negative, which presents a problem because the intercept is written as $\log b_0$. However, from my replication of the results it is seen that the quantity reported is not b_0 but rather $b_0^* = \log b_0$. The signs of the estimates b_1 through b_4 are all as predicted, although b_1 may be larger in magnitude, and b_3 smaller, than would follow from the homogeneity hypothesis. The estimates of a_0 and a_2 imply a ratio of hidden to recorded income in a static economy of 12.6 percent. If the growth rate in real income is assumed to be a steady 2.6 percent per annum (the average for Canada over the sample period) then the ratio of hidden to recorded income is indicated to be 8.8 percent.

With these estimated parameters from the currency demand equation and the actual sample values of recorded real income from 1974 onward as listed in the appendix, we can calculate the time path of hidden income for 1976-1995. From equation 10 we have $\hat{Y}_{Ht} = \hat{a}_0 Y_{Rt} + \hat{a}_2 Y_{Rt-2}$, where $\hat{a}_0 = -0.4618$ and $\hat{a}_2 = 0.5877$. When the calculation is converted to a percentage of recorded GDP, the original benchmarking results are replicated almost exactly.[42]

42 I agree with the reported maximum of 13.80 percent and find an average of 9.459 (which is closer to 9.46 than to 9.45). However, the smallest value in the period is 6.59 percent (which occurs in 1985), not 7.25 percent as reported, which is the second smallest. The time profile looks nothing like the series from the MIMIC model.

Identification in the Currency Demand Model

The replication of benchmarking takes the published estimates of the currency demand model as given. Unfortunately, as I will now show, these "estimates" have little or nothing to do with any evidence in the data. The problem is that the model is not identified, and the parameters used for estimating the underground economy are among those that are indeterminate.

Identification is the requirement that distinct configurations of parameter values can be distinguished empirically, at least in principle, if the right data can be found. Statistically this requires the likelihood function to be different throughout the parameter space. If some value of the parameters is unidentified, then there are other values of the parameters (often infinitely many other values) with different meanings in the model but with exactly the same likelihood function. With the same likelihood function, the mapping from data to parameters that supports one configuration of parameters will equally support others. No amount of any conceivable data can distinguish between these observationally equivalent structures in an unidentified model.

While identification is a logical property of the model, absence of identification has practical consequences for estimation. The most severe will be a failure to produce estimates, because the algorithm for likelihood maximization finds many, equally good, directions to proceed. Different software packages will render the failure differently. Some may simply crash when the going gets tough. Others may be unstable and produce estimates or standard errors that depend on the starting values, perhaps without any warning that termination has been premature. More sophisticated software may be programmed to drop parameters to resolve the ambiguity and to report estimates based on a subset of the parameters. Since there are no established rules for this treatment, different packages may produce different results.

I want to explore the identification of parameters in the currency demand model of Giles and Tedds—without investing in the apparatus of a likelihood function. To this end, the model can be rearranged and the identification question considered in a slightly different parametric form. The first step is to use the properties of logarithms to write equation 7.8 as follows

$$m_t = b_0^* + b_1 \log d_1 + b_1 \log\left(Y_{Rt} + d_2 Y_{Rt-2}\right) + b_2 r_t + b_3 p_t + b_4 r_{t-1} + v_t, \qquad (12)$$

where $d_1 = (1 + a_0)$ and $d_2 = a_2/(1 + a_0)$. Note that the model has not been changed in any material way: it has just been rearranged and some of its components have been relabelled. Importantly, knowing (a_0, a_2) is as good as knowing (d_1, d_2), and vice versa.[43]

Consider equation 12 as if it were a slightly complicated linear regression model. The last three terms will serve to identify their own coefficients b_2, b_3, and b_4 in the

43 The statement is not quite true if $a_0 = -1$ and $a_2 = 0$, but in that case the interpretation of underground economy is nonsense.

usual way. Then the first three terms are required to identify b_0^*, b_1, d_1, and d_2. It is reasonable to expect that b_1 and d_2 can be extracted separately from the nonlinear third term, provided there is enough independent variation over time in real income to ensure that Y_{Rt} is clearly distinguishable from Y_{Rt-2}. But the first two terms are just components of the intercept in this model, which has the composite form $b_0^* + b_1 \log d_1$. Even if one is armed with the value of b_1 from the previous step, it is impossible to solve this composite intercept for the two separate parameters b_0^* and d_1, which indicates that these parameters cannot be separately identified. In terms of the original parameters, since d_1 is unidentified so are both a_0 and a_2. Thus, the parameters essential to forming the benchmark estimate of the underground economy are indeterminate.

An Empirical Demonstration

We expect econometric software to experience problems with this currency demand model, perhaps failing to produce estimates or implementing some mechanical rule to remove the identification problem and thereby enable estimation of the other parameters. If estimates are produced, we expect them to be sensitive to features of the method that should be irrelevant, such as the particular estimation algorithm or the starting values used in the algorithm. In particular, the parameters used to infer the benchmark level of the underground economy will be among the least reliable.

The estimates reported by Giles and Tedds could no doubt be replicated with knowledge of the particular estimation algorithm and starting values that they used. However, the more interesting issue is the problems caused by identification failure in this model. To illustrate empirically, I will employ two software packages and make small perturbations to the starting values. One package is Shazam version 8.0 of 1997, because that was used by Giles and Tedds.[44] I also use Stata version 8.2 of 2004 as an example of a modern, highly regarded, econometrics package. In both of these programs the command for nonlinear regression estimation is called "NL." All options on these commands are set at their defaults.

I find markedly different results from those published. In addition, the results obtained from the two packages vary wildly from each other and show further instability when different initial starting values are provided. The results are recorded in table 2, where column 1 reports the published results.[45] Columns 2 through 5 contain my estimates by Shazam, while columns 6 through 9 contain estimates from Stata. For each package there are four sets of estimates, distinguished by the different starting values given to the algorithm. Below each coefficient estimate is its t-ratio as reported by the package. To show the overall effects of the different experiments, across the bottom of table 2 are recorded the number of iterations taken by the nonlinear algorithm, the residual sum of squares at termination (which determines

44 The Shazam 8.0 manual is referenced in Giles and Tedds, supra note 1, at 141, note 30.

45 Taken from Giles and Tedds, ibid., at 138, table 7.7.

TABLE 2 Re-Estimates of Currency Demand Model[a]

Coeff (t-ratio)	(1) Published table 7.7	(2) Shazam start 0	(3) Shazam start 1	(4) Shazam start 2	(5) Shazam start 3	(6) Stata start 0	(7) Stata start 1	(8) Stata start 2	(9) Stata start 3
b_0^*	−0.3702	−0.3702	−0.5316	−0.3749	−0.2971	−0.3702	−0.3702	−0.2231	−0.6931
	(0.31)	(0.40)	(0.38)	(0.29)	(0.21)	(***)	(***)	(***)	(***)
b_1	1.6587	1.6587	1.6587	1.6587	1.6587	1.6587	1.6587	1.6587	1.6587
	(9.07)	(9.08)	(9.10)	(9.16)	(9.06)	(7.60)	(7.60)	(7.60)	(7.60)
b_2	−0.0751	−0.0751	−0.0751	−0.0751	−0.0751	−0.0751	−0.0751	−0.0751	−0.0751
	(2.71)	(2.75)	(2.73)	(2.75)	(2.74)	(2.31)	(2.31)	(2.31)	(2.31)
b_3	0.4041	0.4041	0.4041	0.4041	0.4041	0.4041	0.4041	0.4041	0.4041
	(3.90)	(3.91)	(3.93)	(3.95)	(3.90)	(3.27)	(3.27)	(3.27)	(3.27)
b_4	−0.1148	−0.1148	−0.1148	−0.1148	−0.1148	−0.1148	−0.1148	−0.1148	−0.1148
	(3.64)	(3.65)	(3.65)	(3.66)	(3.66)	(3.07)	(3.07)	(3.07)	(3.07)
a_0	−0.4618	−0.4618	−0.4068	−0.4603	−0.4850	−0.4618	−0.4618	−0.5074	−0.3461
	(3.37)	(1.04)	(2.34)	(3.07)	(3.11)	(0.97)	(0.97)	(1.18)	(0.58)
a_2	0.5877	0.5877	0.6477	0.5893	0.5623	0.5877	0.5877	0.5378	0.7140
	(4.26)	(1.20)	(3.79)	(3.64)	(3.35)	(1.11)	(1.11)	(1.12)	(1.08)
Iterations		9	26	17	15	1	3	2	3
Residual SS[b]		0.014942	0.014942	0.014942	0.014942	0.014942	0.014942	0.014942	0.014942
Benchmark, %		9.46	20.6	9.77	4.74	9.46	9.46	0.17	33.0

[a] Starting values: Start 0 = estimates as published.
Start 1 = b_0^* as published, $b_1 = b_3 = 1.0$, $b_2 = b_4 = -0.1$, $a_0 = -0.5$, $a_2 = 0.6$.
Start 2 = as published, except b_0^* varied to $\log(0.8) = -0.2231$.
Start 3 = as published, except b_0^* varied to $\log(0.5) = -0.6931$.

[b] *SS* = sum of squares.

the value of the likelihood function), and the benchmark value for the underground economy obtained from estimates of a_0 and a_2 by the method described previously.

To check the setup—and let the reported results make their best showing—I begin with the published estimates as the starting values. Hence the initial values fed to both Shazam and Stata for this first exercise are −0.3702 for b_0^* and 1.6587 for b_1, etc. These results are reported under "Start 0" in columns 2 and 6 of table 2. Both Shazam and Stata replicate all of the coefficients very closely, confirming that the reported estimates are a stationary point of the likelihood function. Shazam takes nine iterations to converge while Stata requires only one, indicating the comparative efficiency of the two algorithms. Looking at the t-ratios for b_1 through b_4, we see that Shazam is uniformly more optimistic than Stata about significance of estimates, presumably reflecting the different ways that standard errors can be validly obtained in nonlinear estimation. Since the estimates of a_0 and a_2 are the same in both cases, and the same as the initial values, the calculation of the benchmark is 9.46 percent, as it is from the published estimates.

One remarkable difference is the t-ratios on the key parameters a_0 and a_2. Although Shazam is again slightly more optimistic than Stata about significance, in neither package is either of these two parameters remotely significantly different from zero at the usual levels. This contrasts strongly with the t-values stated in the published results of 3.37 and 4.26, respectively, which the authors find "encouraging."[46] I cannot account for this discrepancy, except as an example of the wild variability of estimation results in an unidentified model.

The other contrast in the estimates in columns 2 and 6 is the t-ratio on b_0^*. Both Shazam and Stata return the starting value as the coefficient estimate, and Shazam gives a standard error and therefore a t-ratio to go with the coefficient. But Stata does not provide a standard error, and hence there is no t-ratio on this coefficient. In this model it is theoretically impossible to estimate all three of a_0, a_2, and b_0^* without some extraneous information. Stata's response, quite sensibly, is to fix b_0^* at its initial value and to estimate the other two. Shazam somehow pretends that all three parameters are estimated and quotes a standard error for each of them.

Next we look at the effect of perturbing the starting values slightly. First, b_0^* is kept fixed at the published estimate of −0.3702 and all the others changed to values in the plausible range not too far from the published estimates. These results are reported under "Start 1" in columns 3 and 7. In this case, Stata robustly converges to the same estimates as before, taking just three iterations to do so. Shazam, however, is upset by the change in initial values and produces a completely different set of estimates for the three unidentified parameters. These new estimates are equally valid because the likelihood value is equally maximized at this point. But now the implications for the benchmark are completely different: the new coefficients imply a benchmark estimate of the underground economy at 20.6 percent of GDP. This time the t-ratios reported by Shazam on the important estimates are indeed "encouraging."

46 Ibid., at 137.

The other four columns in table 2 record experiments where only b_0^* is varied from its starting value of $-0.3702 \approx \log(0.7)$, and all the other parameters are started from their published estimates. Under "Start 2" in columns 4 and 8, the variation is up slightly to $\log(0.8) = -0.2231$, and under "Start 3" in columns 5 and 9 the variation is down slightly to $\log(0.5) = -0.6931$. As with the previous perturbation of the starting values, Shazam needs more iterations to achieve convergence than before and many more than Stata. Stata recognizes the lack of identification, holds b_0^* at its initial value, and estimates the other parameters conditional on that value. Shazam, by contrast, presents estimates for all of the parameters. But the arbitrariness of the "estimation" solution in all cases is evident from the variation in the estimates of a_0 and a_2 across the experiments. The implied benchmark of the underground economy in Canada varies from 4.74 to 20.6 percent of GDP in the estimates from Shazam, and from 0.17 percent to 33.0 percent in the estimates from Stata. Even more dramatic variation can be achieved with larger perturbations of the initial conditions. In this light, it is difficult to accept the authors' assurance that "[t]he results in the table are robust with respect to the choice of starting values for the non-linear estimation algorithm."[47]

CONCLUSIONS

Giles and Tedds estimate an underground economy in Canada that is remarkably large and growing rapidly. Their time series of hidden economic activity as a percentage of measured GDP rises from 3.46 percent in 1976 to 15.64 percent in 1995. However, I can account for their findings by features of the modelling and data that have little or nothing to do with their interpretation of the series.

Understanding their results requires peeling away the layers of econometric complication, which include MIMIC modelling, prediction, and benchmarking. I am able to show that the overall time pattern of their series, including its surprising growth by a factor of 4.5 over 20 years, is attributable to their prediction formula. Their index of underground economic activity is derived almost entirely from a single variable—namely, the declared incomes of self-employed persons. This variable is an economy-wide aggregate and measured in nominal dollars. Naturally, it experiences strong growth from the mid-1970s to the mid-1990s, in part due to real growth of the Canadian economy but mostly due to price inflation. There is nothing in such a variable to sustain its interpretation as an index of underground economic activity relative to observed GDP.

The overall level of the index is established by a benchmark from a separate currency demand model. This model is unidentified, so the "estimates" of its key parameters are merely numerical accidents without connection to the data. I show

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47 Ibid. My findings are more consistent with this revised view of one of the authors: "This model, however, has subsequently been found to be extremely unstable. In particular, the highly nonlinear model that was used in the previous work proved to be sensitive to small data changes." Tedds, supra note 17, at 11 and note 8.

that a wide variety of benchmark values can be obtained by tweaking the estimation method in ways that should have no effect.

This leaves the question of why the numbers have any plausibility at all: why not an estimate of the underground economy in Canada that is negative or larger than 100 percent? The answer must lie in the control that researchers exercise over their methods to ensure that the results are interesting and reasonable (meaning challenging but not too outlandish). Quite rightly, Giles and Tedds are careful to place their results in the context of the literature, particularly the existing estimates for Canada. They cite the similarity of their results to other findings (at least on average) as validating their methodology, while promoting the novelty of their 20-year time path. But it seems that their published results are chosen from among a wide spectrum of equally valid possibilities, not just by reference to the evidence in their data and the internal logic of their modelling, but also on an understanding of what numbers are interesting and reasonable.

While this paper has been limited to Giles and Tedds's book and their results for Canada, similar methods are being adopted by other researchers and applied to other countries. These other applications will become the subject of scrutiny in future research, as sufficient details of their inner workings become available. In part the criticism falls on Giles and Tedds because they are pioneers in the methods. It is also unfortunate that they are vulnerable because they document their calculations and provide their data for others to analyze. In this openness and accountability they set a high standard for scientific behaviour in economics that is not always followed.

APPENDIX

TABLE A1 Variable Names and Descriptions

Indicator variables:

CURR	Currency (notes and coins) in circulation outside of banks
GDP	Real gross domestic product (in 1986 dollars)

Causal variables:

MULT	Total number of male holders of multiple jobs aged 15 or older
SELF	Income of self-employed persons
INC	Real annual disposable income per member of the labour force
ERTE	Nominal Canadian-US exchange rate (Cdn $/US $)
CORP	Ratio of total corporate tax revenue to GDP
IND	Ratio of total indirect tax revenue to GDP
UNEM	Unemployment rate

Currency demand, other variables:

IRATE	Bank of Canada bank rate, end of year
NGDP	Nominal GDP (= GDP × IPD)
IPD	Implicit GDP deflator, base = 1 in 1986

Source: David E.A. Giles and Lindsay M. Tedds, *Taxes and the Canadian Underground Economy*, Canadian Tax Paper no. 106 (Toronto: Canadian Tax Foundation, 2002), 129, table 7.2 and 137, table 7.6.

TABLE A2 Causal Variables for MIMIC Model

Year	MULT	SELF	INC	ERTE	CORP	IND	UNEM
1976	207917	3491691	25616.95	0.986028	3.601382	12.46236	7.2
1977	240333	3570357	25345.55	1.063440	3.322027	12.40459	8.1
1978	252417	3927510	25310.64	1.140660	3.389017	11.93523	8.4
1979	309250	4352728	25326.08	1.171420	3.635692	11.63074	7.5
1980	345667	4846537	25268.38	1.169230	3.897499	11.45725	7.5
1981	383167	5449589	25474.06	1.198900	3.594443	12.90921	7.6
1982	356917	6103106	25314.00	1.233730	3.139338	12.88531	11.0
1983	387333	7199871	24686.99	1.232410	3.036599	12.36083	11.9
1984	406667	8252571	25262.69	1.295070	3.369197	12.35725	11.3
1985	436750	9182331	25480.11	1.365510	3.255940	12.29926	10.5
1986	464667	9762675	25251.83	1.389470	2.881942	12.72342	9.6
1987	510250	10029293	25379.10	1.325980	3.080147	12.93789	8.9
1988	575917	11759114	26085.23	1.230700	2.902430	13.04328	7.8
1989	602083	13468663	26762.75	1.183970	2.853025	13.34895	7.5
1990	635667	14296028	26391.83	1.166770	2.516031	13.08504	8.1
1991	615500	14674888	25624.32	1.145730	2.219588	13.84171	10.4
1992	621167	15038195	25769.12	1.208720	2.103541	14.16300	11.3
1993	656583	15401501	25534.70	1.290090	2.050627	14.02487	11.2
1994	653333	16273783	25548.55	1.365670	2.341488	13.79908	10.4
1995	658500	16435297	25498.71	1.372450	2.597195	13.52520	9.5

Source: Data supplied by Lindsay Tedds.

TABLE A3 Other Variables: Indicators and Currency Demand Model

Year	GDP	CURR	IRATE	NGDP	IPD
1974	341235	5791	8.75	152111	0.445766
1975	350113	6748	9.00	171540	0.489956
1976	371688	7241	8.50	197924	0.532500
1977	385122	7970	7.50	217879	0.565740
1978	402737	8755	10.75	241604	0.599905
1979	418328	9460	14.00	276096	0.659999
1980	424537	10263	17.26	309891	0.729951
1981	440127	10596	14.66	355994	0.808844
1982	425970	11392	10.26	374442	0.879034
1983	439448	12401	10.04	405717	0.923242
1984	467167	13364	10.16	444735	0.951983
1985	489437	14379	9.49	477988	0.976608
1986	505666	15455	8.49	505666	1.000000
1987	526730	16652	8.66	551597	1.047210
1988	552958	17932	11.17	605906	1.095754
1989	566486	19107	12.47	650748	1.148745
1990	565576	19776	11.78	670952	1.186316
1991	555052	21050	7.67	676477	1.218763
1992	559305	22790	7.36	690122	1.233892
1993	571722	24400	4.11	712855	1.246856
1994	594990	26152	7.43	747260	1.255920
1995	608835	26835	5.79	776299	1.275056

Source: Data supplied by Lindsay Tedds.

More Formal Proof of Identification Results

The estimation model 7.8 is a single-equation, nonlinear regression with additive Gaussian errors, in the generic form

$$y_t = x_t(\beta) + \varepsilon_t, \quad \varepsilon_t \text{ is } NID(0, \sigma^2), \quad \text{for } t = 1, \dots, N, \tag{A1}$$

where $x_t(\beta)$ is the regression function and β is a vector of parameters. An elegant analytical framework for a whole host of identification and estimation issues in this class of models is the Gauss-Newton regression. Given some initial value $\overline{\beta}$, the Gauss-Newton regression is the local linear model

$$\overline{y}_t = X_t(\overline{\beta})b + \varepsilon_t, \tag{A2}$$

where $b = \beta - \overline{\beta}$, $\overline{y}_t = y_t - x_t(\overline{\beta})$ and $X_t(\overline{\beta}) = \partial x_t(\beta)/\partial\beta$ evaluated at $\overline{\beta}$. In matrix notation for all $t = 1, \dots, N$ observations, the initial value can be updated by the familiar least squares formula for linear regression

$$\hat{b} = (\overline{X}'\overline{X})^{-1}\overline{X}'\overline{y}. \tag{A3}$$

The Gauss-Newton algorithm for estimating a nonlinear regression model consists of repeated application of this formula until there is no more change. In practice, well-designed software to implement the algorithm will have safeguards to assist convergence.

Davidson and MacKinnon[48] use the Gauss-Newton regression to show that many well-known results for linear regression carry over to the more complicated setting of the nonlinear regression model. In particular, an identification breakdown in the nonlinear model is equivalent to collinearity in the Gauss-Newton regression, which will be evident from linear relationships among the columns of $X = X(\beta)$.

Consider now the particular case of the model in 7.8. Collect the parameters in the regression function of this model into the vector

$$\beta = \left(b_0^*,\, b_1,\, b_2,\, b_3,\, b_4,\, a_0,\, a_2\right)'. \qquad \text{(A4)}$$

The derivatives of the nonlinear regression function with respect to the parameters are then

$$X_t(\beta) = \left[1,\, \log G_t,\, r_t,\, p_t,\, r_{t-1},\, \frac{b_1 Y_{Rt}}{G_t}, \frac{b_1 Y_{Rt-2}}{G_t}\right], \qquad \text{(A5)}$$

where $G_t = [(1 + a_0)Y_{Rt} + a_2 Y_{Rt-2}]$. I will assume that there is some variation in the growth rate of recorded real income across the sample, so that Y_{Rt} is not collinear with Y_{Rt-2}. This assumption simply rules out a trivial identification problem; I note that the data are sufficiently variable in this way. I also assume that the parameters do not take any of the isolated values that make G_t a scaled version of Y_{Rt} (this would be a result of $a_2 = 0$) or of Y_{Rt-2} (a result of $a_0 = -1$). These are also ways the columns of X will be collinear, and hence $X'X$ will be singular and the model not identified; but for the present purpose they are uninteresting special cases. I will focus on a quite general problem that pertains to any point in the parameter space of this model.

Given any values for b_1, a_0, and a_2, if the sixth column of X is multiplied by $(1 + a_0)$ and added to the seventh (last) column multiplied by a_2, the result is identical to the first column multiplied by b_1. This indicates that these three columns are collinear. Hence the parameters that correspond to these columns, namely a_0, a_2, and b_0^* (or equivalently, b_0), cannot be separately identified. The collinearity is exact, and it holds globally throughout the parameter space.

48 Russell Davidson and James G. MacKinnon, *Estimation and Inference in Econometrics* (New York: Oxford University Press, 1993).